Théorie de la synchronisation - Equation de Duffing

Etude du régime transitoire - Oscillateur en avance critique sur l'excitation

Caractéristiques du système

$T := 0.2 \cdot s \qquad \omega_0 := \frac{2 \cdot \pi}{T} \qquad J := 8 \cdot 10^{-7} \cdot kg \cdot m^2 \qquad q_0 := 270 \cdot deg$

Frottement visqueux

$\eta := 0.002 \qquad C := 2 \cdot J \cdot \eta \cdot \omega_0 \qquad F_{v\_max} := C \cdot \omega_0 \cdot q_0 \qquad \lambda := \frac{F_{v\_max}}{J \cdot \omega_0^2} \qquad h := 2 \cdot \frac{\eta}{\lambda}$

$F_{v\_max} = 0.015\, N \cdot mm \qquad \lambda = 0.019 \qquad h = 0.212$

Frottement quadratique

$B := 0.05 \cdot F_{v\_max} \qquad \beta_1 := \frac{B}{\lambda \cdot J \cdot \omega_0^2} \qquad \beta_1 = 0.05 \qquad F_{q\_max} := B \cdot q_0^3 \qquad F_{q\_max} = 0.078\, N \cdot mm$

Régimes transitoires vers un foyer attractif

Excitation harmonique

$A_c := \frac{8 \cdot h^3}{3\sqrt{3}} \qquad A_c = 0.015 \qquad \boxed{A := .5 \cdot A_c} \qquad A = 7.356 \times 10^{-3}$

$a_1 := \sqrt{\frac{4 \cdot A}{3 \cdot \beta_1}} \qquad F_{harm} := a_1 \cdot \left(\lambda \cdot J \cdot \omega_0^2\right) \qquad F_{harm} = 6.592 \times 10^{-6}\, N \cdot m$

$n := 500 \qquad i := 0 .. n \qquad x_0 := 0 \qquad x_1 := 2 \cdot \pi \qquad \Delta x := \frac{x_1 - x_0}{n} \qquad x_i := x_0 + i \cdot \Delta x$

$Y_i := \frac{a_1}{h} \cdot \sin(x_i) \cdot (0 < x_i < \pi) \qquad \varepsilon := \sqrt{3} \cdot h \qquad \sqrt{3} = 1.732 \qquad \boxed{\varepsilon = 0.368}$

$$X := \left[\begin{array}{l} \text{for } i \in 0..n \\ \quad Z \leftarrow polyracines\left(\left(-a_1 \cdot \cos(x_i) \quad \varepsilon \quad 0 \quad \frac{-3}{4} \cdot \beta_1\right)^T\right) \\ \quad \text{for } j \in 0..2 \\ \qquad X_{i,j} \leftarrow Z_j \cdot \left(Im(Z_j) = 0\right) \cdot \left(Re(Z_j) > 0\right) \\ X \end{array}\right]$$

$v := \left(-A \quad \varepsilon^2 + h^2 \quad -2 \cdot \varepsilon \quad 1\right)^T \qquad Z := polyracines(v) \qquad y_s := \sqrt{\frac{4 \cdot Z}{3 \cdot \beta_1}} \qquad x_s := \arccos\left[\frac{1}{a_1} \cdot \left(\varepsilon \cdot y_s - \frac{3}{4} \cdot \beta_1 \cdot y_s^3\right)\right]$

$$\frac{Z}{h} = \begin{pmatrix} 0.238 \\ 1.613 - 0.794i \\ 1.613 + 0.794i \end{pmatrix} \qquad y_s = \begin{pmatrix} 1.161 \\ 3.106 - 0.723i \\ 3.106 + 0.723i \end{pmatrix} \qquad x_s = \begin{pmatrix} 0.59 \\ 1.276 - 1.011i \\ 1.276 + 1.011i \end{pmatrix}$$

$$Y0 := \begin{pmatrix} 2.5 \\ 0.4 \end{pmatrix} \qquad \mathbf{D}(t, Y) := \begin{bmatrix} \left(\frac{-h}{2} \cdot Y_0 + \frac{a_1}{2} \cdot \sin(Y_1)\right)\lambda \\ \lambda \cdot \left[\frac{-\varepsilon}{2} + \frac{3}{8} \cdot \beta_1 \cdot (Y_0)^2 + \frac{a_1}{2 \cdot Y_0} \cdot \cos(Y_1)\right] \end{bmatrix}$$

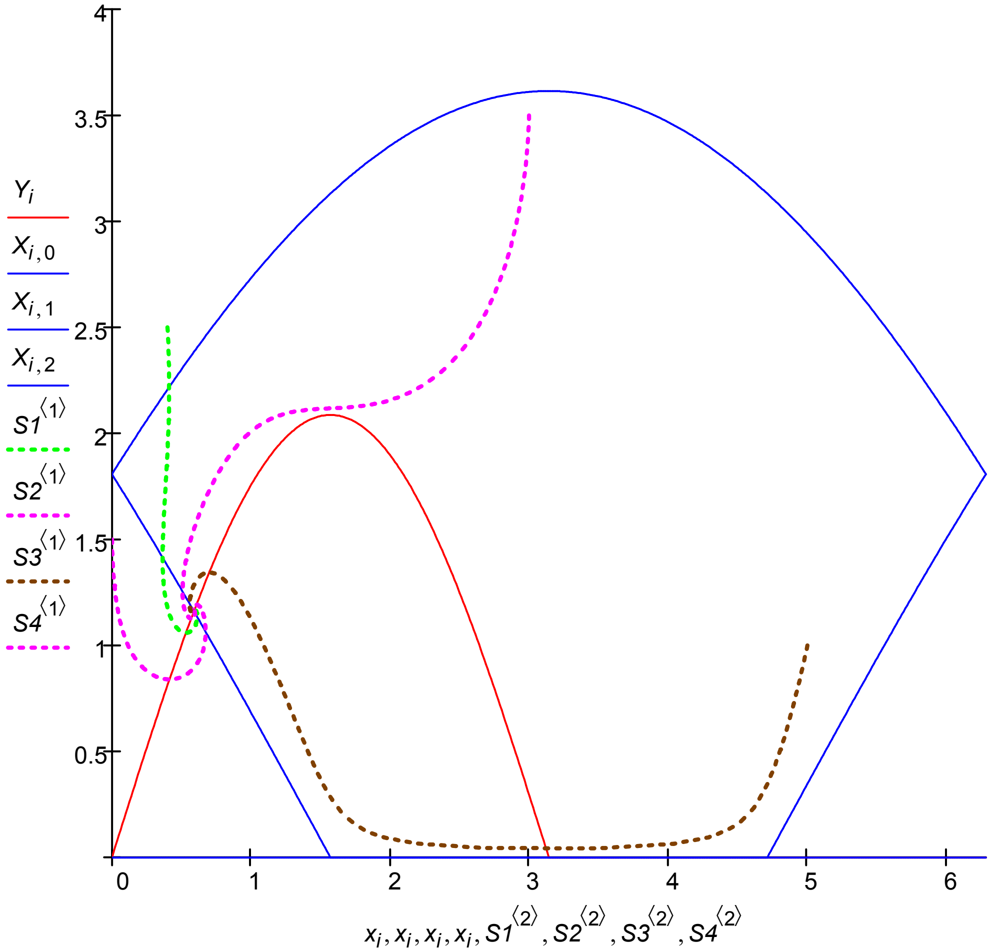
$t_f := 4000$

$Y0 := \begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$ $\quad S1 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\quad S1^{\langle 2 \rangle} := mod\left(S1^{\langle 2 \rangle}, 2\cdot\pi\right)$

$Y0 := \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ $\quad S2 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\quad S2^{\langle 2 \rangle} := mod\left(S2^{\langle 2 \rangle}, 2\cdot\pi\right)$

$Y0 := \begin{pmatrix} 3.5 \\ 3 \end{pmatrix}$ $\quad S3 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\quad S3^{\langle 2 \rangle} := mod\left(S3^{\langle 2 \rangle}, 2\cdot\pi\right)$

$S4 := rkfixe(Y0, 0, t_f, n, \mathbf{D})$ $\quad S4^{\langle 2 \rangle} := mod\left(S4^{\langle 2 \rangle}, 2\cdot\pi\right)$



Régimes transitoires structurellement instables

Excitation harmonique

$\boxed{A := 1\cdot A_c}$ $\quad A = 0.015$ $\quad \varepsilon := \sqrt{3}\cdot h$ $\quad \boxed{\varepsilon = 0.368}$

$a_1 := \sqrt{\dfrac{4\cdot A}{3\cdot \beta_1}}$ $\quad F_{harm} := a_1\cdot\left(\lambda\cdot J\cdot \omega_0^{\ 2}\right)$ $\quad F_{harm} = 9.322\times 10^{-6}\, N\cdot m$ $\quad \sqrt{3} = 1.732$

$Y_i := \dfrac{a_1}{h}\cdot sin(x_i)\cdot(0 < x_i < \pi)$

$$X := \left| \begin{array}{l} \text{for } i \in 0..n \\ \quad \left| \begin{array}{l} Z \leftarrow polyracines\left(\left(-a_1\cdot cos(x_i) \quad \varepsilon \quad 0 \quad \dfrac{-3}{4}\cdot\beta_1\right)^T\right) \\ \text{for } j \in 0..2 \\ \quad X_{i,j} \leftarrow Z_j\cdot\left(Im(Z_j) = 0\right)\cdot\left(Re(Z_j) > 0\right) \end{array} \right. \\ X \end{array} \right.$$

$v := \left(-A \quad \varepsilon^2 + h^2 \quad -2\cdot\varepsilon \quad 1\right)^T$ $\quad Z := polyracines(v)$ $\quad y_s := \sqrt{\dfrac{4\cdot Z}{3\cdot\beta_1}}$ $\quad x_s := arccos\left[\dfrac{1}{a_1}\cdot\left(\varepsilon\cdot y_s - \dfrac{3}{4}\cdot\beta_1\cdot y_s{}^3\right)\right]$

$$\frac{Z}{h} = \begin{pmatrix} 1.155 \\ 1.155 \\ 1.155 \end{pmatrix} \qquad y_s = \begin{pmatrix} 2.556 \\ 2.556 \\ 2.556 \end{pmatrix} \qquad x_s = \begin{pmatrix} 1.047 \\ 1.047 \\ 1.047 \end{pmatrix}$$

$x_{min} := arccos\left(\dfrac{4}{9}\cdot\dfrac{\varepsilon}{a_1}\cdot\sqrt{\dfrac{\varepsilon}{\beta_1}}\right)$

$x_{max} := 2\cdot\pi - x_{min}$

$x_{min} = 0.785 \qquad x_{max} = 5.498$

$$Y0 := \begin{pmatrix} 2.5 \\ 0 \end{pmatrix} \qquad \boldsymbol{D}(t, Y) := \begin{bmatrix} \left(\dfrac{-h}{2}\cdot Y_0 + \dfrac{a_1}{2}\cdot sin(Y_1)\right)\lambda \\ \lambda\cdot\left[\dfrac{-\varepsilon}{2} + \dfrac{3}{8}\cdot\beta_1\cdot(Y_0)^2 + \dfrac{a_1}{2\cdot Y_0}\cdot cos(Y_1)\right] \end{bmatrix}$$

$t_f := 4000 \qquad S1 := rkfixe(Y0, 0, t_f, n, \boldsymbol{D}) \qquad S1^{\langle 2\rangle} := mod\left(S1^{\langle 2\rangle}, 2\cdot\pi\right)$

$Y0 := \begin{pmatrix} 1.5 \\ 0 \end{pmatrix} \qquad S2 := rkfixe(Y0, 0, t_f, n, \boldsymbol{D}) \qquad S2^{\langle 2\rangle} := mod\left(S2^{\langle 2\rangle}, 2\cdot\pi\right)$

$Y0 := \begin{pmatrix} 1 \\ 5 \end{pmatrix} \qquad S3 := rkfixe(Y0, 0, t_f, n, \boldsymbol{D}) \qquad S3^{\langle 2\rangle} := mod\left(S3^{\langle 2\rangle}, 2\cdot\pi\right)$

$Y0 := \begin{pmatrix} 3.5 \\ 3 \end{pmatrix} \qquad S4 := rkfixe(Y0, 0, t_f, n, \boldsymbol{D}) \qquad S4^{\langle 2\rangle} := mod\left(S4^{\langle 2\rangle}, 2\cdot\pi\right)$